Register No : ____________________

# THE KAVERY ENGINEERING COLLEGE

*(An Autonomous Institution)*

MCA DEGREE END SEMESTER THEORY EXAMINATIONS, NOV /DEC 2024

*Third Semester*

*Master of Computer Applications*

BX4005 - Mathematical Foundations of Computer Science

*Regulations - 2021*

*Duration : 3 Hrs* *Maximum : 100 Marks*

*PART – A (ANSWER ALL QUESTIONS) (Marks 10*2=20)*

| Q.No | Questions | BLT | CO | Marks |
|---|---|---|---|---|
| 1. | Write the negation of the quantified statement "Every student has at least one course where the lecturer is a teaching assistant". | UN | 1 | 2 |
| 2. | Let $L(x,y)$ be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express, "Joy is loved by everyone." | UN | 1 | 2 |
| 3. | Let $P(n)$ be "$8^n - 3^n$ is a multiple of 5". Prove that $P(n)$ is a Tautology over N. | UN | 2 | 2 |
| 4. | Find the recurrence relation for $D(K) = 2K + 7$ | UN | 2 | 2 |
| 5. | Define simple graph with example. | RE | 3 | 2 |
| 6. | Show that the maximum number of edges in a simple graph with a vertices is $\frac{n(n-1)}{2}$. | UN | 3 | 2 |
| 7. | Show that $(N,+)$ is a semi group but not a monoid. | UN | 4 | 2 |
| 8. | Define abelian group. | RE | 4 | 2 |
| 9. | Define partially ordered set. | RE | 5 | 2 |
| 10. | Define Boolean algebra. | RE | 5 | 2 |

*PART – B (ANSWER ALL QUESTIONS) (Marks 5*13 = 65)*

| Q.No | | | Questions | BLT | CO | Marks |
|---|---|---|---|---|---|---|
| 11. | a | i | Show that $S \vee R$ tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$. | UN | 1 | 7 |
| | | ii | Determine the truth value of each of the following statements where $U = \{1,2,3\}$ is the universal set:<br>a. $\exists x \forall y, x^2 < y+1,$<br>b. $\forall x \exists y, x^2 + y^2 < 12,$<br>c. $\forall x \forall y, x^2 + y^2 < 12,$ | UN | 1 | 6 |
| | | | Or | | | |
| | b | i | Show that $P \to S$ can be derived from the premises, $\neg P \vee Q, \neg Q \vee R,$ and $R \to S$. | UN | 1 | 7 |
| | | ii | Convert $(\forall x)(\forall y)(\forall z)(A(x,y,z) \wedge B(y)) \to (\forall x)C(x,z))$ to prenex normal form. | UN | 1 | 6 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 12. | a | i | A salesman sells at least one car each day for 100 consecutive days selling a total of 150 cars. Show that for each value of n with $1 \le n < 50$, there is a period of consecutive days during which he sold atotal of exactly n cars. | AP | 2 | 6 |
| | | ii | How many five digit even numbers can be formed using the figures 0, 1, 2, 3, 5, 7 and 8 without using a figure more than once in a number? | AP | 2 | 7 |
| | | | Or | | | |
| | b | | Solve the recurrence relation of the Fibonacci sequence of numbers. | AP | 2 | 13 |
| 13. | a | i | A simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges. | UN | 3 | 7 |
| | | ii | Show that the complete bi-partite graph $K_{3,3}$ is not planar graph. | UN | 3 | 6 |
| | | | Or | | | |
| | b | | Let G be a linear graph of n vertices. If the sum of the degrees for each pair of vertices in G in $n-1$ or larger, then there exists a Hamiltonian path in G. | UN | 3 | 13 |
| 14. | a | i | Show that the set $M_{2\times 2}$ of all $2\times 2$ non – singular matrices over R (reals) is a group but not abelian group under usual matrix multiplication. | UN | 4 | 7 |
| | | ii | Show that the kernel of a homomorphism $f: G \to H$ is a normal subgroup of G. | UN | 4 | 6 |
| | | | Or | | | |
| | b | | State and Prove Lagrange's theorem. | UN | 4 | 13 |
| 15. | a | i | Show that both LUB and GLB of any elements in a poset are unique. | AP | 5 | 6 |
| | | ii | Show that the set of integers which are divisors of 60 is partially ordered set. Also draw its Hasse diagram. | AP | 5 | 7 |
| | | | Or | | | |
| | b | i | In a distributive lattice, show that $(a*b)\oplus(b*c)\oplus(c*a)=(a\oplus b)*(b\oplus c)*(c\oplus a)$. | AP | 5 | 6 |
| | | ii | Find the sum-of-products and product-of-sum of the following Boolean expression $(x\oplus y)*(x'\oplus z)$. | AP | 5 | 7 |

*PART – C (Marks 1*15 = 15)*

| *Q.No* | | *Questions* | *BLT* | *CO* | *Marks* |
|---|---|---|---|---|---|
| 16. | a | There are 100 people in a certain room. In this group, 60 men, 30 are young, and 10 are young men. How many are old women? | AP | 2 | 15 |
| | | Or | | | |
| | b | Using generating function, solve the difference equation $S(n+2)-6S(n+1)+5S(n)=0$, given that $S(0)=1$ and $S(1)=2$. | AP | 2 | 15 |